Register No : ____________________

THE KAVERY ENGINEERING COLLEGE

(An Autonomous Institution)

B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, APRIL-MAY 2025

Fourth Semester

Biomedical Engineering

MA3355 - Random Processes and Linear Algebra

(Common to Third Semester Electronics and Communication Engineering)

Statistical Table containing standard normal table Can be Permitted

Regulations - 2021

Duration : 3 Hrs ***Maximum : 100 Marks***

*PART – A (ANSWER ALL QUESTIONS) (Marks 10*2=20)*

Q.No	*Questions*	*BLT*	*CO*	*Marks*
1.	In a random experiment, A and B are the two events such that P(A) = 1/12, P(B) = 5/12 and P(B|A)=1/15, then find P (A ∪ B).	L2	1	2
2.	For a binomial distribution mean is 6 and variance is 2, find the number of trials n of the distribution.	L2	1	2
3.	A function of the random variable (X,Y) is given by $f(x,y) = k(6-x-y); 0<x<2; 2<y<4$. For what value of the constant k, the function f(x,y) is the joint probability density function?	L2	2	2
4.	State the Central Limit Theorem for Identically Independent random variables.	L1	2	2
5.	Define a stationary process.	L1	3	2
6.	State the postulates of a Poison Process.	L1	3	2
7.	Define subspace.	L1	4	2
8.	Determine whether $(-2,0,3)$ can be expressed as a linear combination of (1,3,0) and (2,4, –1).	L2	4	2
9.	Determine the norm and distance between the vectors $u=(1,0,1)$ and $v=(-1,1,0)$.	L2	5	2
10.	Define adjoint operator.	L1	5	2

*PART – B (ANSWER ALL QUESTIONS) (Marks 5*16 = 80)*

Q.No			*Questions*	*BLT*	*CO*	*Marks*
11.	a		A random variable X has the following distribution	L3	1	16

X	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find the value of k, $P(X<6)$, $P(X \geq 6), P(1 \leq X \leq 4)$.

Or

	b	i	Given the probability density function of a triangular distribution	L3	1	8

$$f(x)=\begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean, variance and the moment generating function.

		ii	An electrical firm manufactures light bulbs that have a life before burns-out, is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find (i)The probability that a bulb burns more than 834 hours. (ii) The probability that a bulb burns between 778 and 834 hours.	L3	1	8
12.	a		The joint probability mass function of (X,Y) is given by, $P(x,y) = K(2x+3y)$, x=0,1,2; y=1,2,3. Find (i) the constant K (ii) the marginal distribution of X and Y (iii) all the conditional probability distributions (iv)$P(X \geq 1, Y \leq 2)$ (v) distribution of X+Y (vi) P[X+Y>3]	L3	2	16
			Or			
	b	i	The joint pdf of X and Y is given by $f(x,y) = 4xye^{-(x^2+y^2)};\ \ x \geq 0, y \geq 0$ Find the density function of $U = \sqrt{x^2+y^2}$.	L3	2	8
		ii	A sample of size 100 taken from a population whose mean is 60 and variance is 400. Using Central Limit Theorem, with what probability can we infer that the mean of the sample will not differ from $\mu = 60$ by more than 4. Given the area under the standard normal curve form 0 to 2 is 0.4772.	L3	2	8
13.	a	i	Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (1) Exactly 4 customers arrive (2) more than customers 4 arrive (3) fewer than 4 customers arrive	L3	3	8
		ii	A salesman territory consists of three cities A, B, C. He never sells in the same city on successive days. If he sells in A, then the next day he sells in City B. However if he sells in either B or C then the next day he is twice as likely to sell in city A as in the other city. In the long run, how often does he sell in each of the cities?	L3	3	8
			Or			
	b	i	The TPM of the Markov chain x_n having 3 states 1,2 and 3 is P= $\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7\ \ 0.2\ \ \ 0.1)$. Find (a) $P\{x_3 = 3, x_2 = 3, x_1 = 3, x_0 = 2\}$ (b) $P\{x_2 = 3\}$	L3	3	8

		ii	The process {X(t)} whose probability distribution under certain conditions is given by: $$P\{X(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots\\ \dfrac{at}{1+at}, & n=0\end{cases}$$ Verify whether the process is a stationary process or not.	L3	3	8
14.	a	i	Verify whether the first polynomial can be expressed as a linear combination of the other two in P3(R) for the given x^3-8x^2+4x , x^3-2x^2+3x-1 and x^3-2x+3.	L2	4	8
		ii	Let W_1 and W_2 be subspaces of V. Prove that W_1 U W_2 is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.	L2	4	8
			Or			
	b	i	Let S = {v_1,v_2,v_3} where v_1=(1,–3, –2), v_2=(–3,1,3), v_3=(–2, –10, –2). Verify whether S forms a basis or not.	L2	4	8
		ii	Determine the given set in P4(R) is linearly dependent or linearly independent for $x^4-x^3+5x^2-8x+6$, $-x^4+x^3-5x^2+5x-3$, x^4+3x^2-3x+5 and $2x^4+x^3+4x^2+8x$	L2	4	8
15.	a	i	Let for T : R3→R2 defined by T(a_1,a_2,a_3) = $(a_1-a_2,2a_3)$. Find the bases for N(T) and compute the nullity of T.	L3	5	8
		ii	Find the matrix representation of the linear transformation $T:V_3(R)\to V_3(R)$ given by $T(a,b,c)=(3a,a-b,2a+b+c)$ with respect to the standard basis $\{e_1,e_2,e_3\}$.	L3	5	8
			Or			
	b		Let the vector space P2 have the inner product $<p,q>=\int_0^1 p(x)q(x)dx$ Apply the Gram-Schmidt process to transform the basis S={u_1,u_2,u_3}={1,x,x^2} into an orthonormal basis.	L3	5	16